The value of public information in vertically differentiated markets*

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Abstract

We show that public information about vertically-differentiated products increases expected vertical differentiation and softens competition. Depending on the efficiency of the market equilibrium, there may be over- or under-investment in public information generation: firms overinvest (underinvest) in information generation if the deadweight loss in the market equilibrium is high (low). The deadweight loss depends on the distribution of consumers' valuations. Because information generation has a positive externality among firms, coordination (e.g. via industry associations) increases information generation. When product qualities are endogenous, information generation may substitute for quality degradation and thus have an additional social benefit.

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1 Introduction

Do market equilibrium outcomes provide participants with efficient incentives to generate public information on the goods traded? This question is of considerable interest from both a regulatory and a researcher's point of view. We contribute to answering it by examining the case of a competitive market with vertically differentiated products, in which product qualities are uncertain but can be partially learned via public signals. These public signals can take the form of reviews by experts in the media (examples are *Consumer Reports* in the US or *Which?* in the UK), quality tests and certification by professional agencies (e.g., rating agencies for financial products, TÜV for industrial goods), industry competitions or trade shows. They are often commissioned by firms, which can invite experts to test their products or set up industry bodies for quality certification.

We use a canonical model of price competition with vertically differentiated products (as in Shaked and Sutton, 1982) augmented by the possibility that, before setting their prices, firms can generate costly public signals correlated with the quality of their products. Better information will have a positive effect on social welfare, because it facilitates the assignment of heterogenous goods to heterogenous agents. We show that in a duopoly this will, however, be accompanied by a price effect: drawing a signal correlated with the quality of any product increases the expected quality distance between quality leader and follower. This will raise expected equilibrium prices, firms' profits, and the deadweight loss generated in the pricing equilibrium. This observation has a number of intriguing implications.

First, in equilibrium firms may under- or overinvest in information generation relative to the social optimum. Which case will occur depends on the market shares of quality leader and follower. We show that under mild restrictions (log concavity, full support, and a sufficiently high minimum taste for quality) these market shares do not depend on the information generated by the firms, but only on the distribution of valuations in the population. When the market share of the quality follower is sufficiently large, there is a substantial deadweight loss in the pricing equilibrium. This loss strictly increases in the amount of information generated and lowers the social benefit of information. Therefore firms overinvest in information generation relative to the social optimum. Conversely, when few consumers purchase from the quality follower, the deadweight loss is small and so is its sensitivity to new information. The social benefit of information generation thus exceeds its private benefit and firms will underinvest in information generation relative to the social optimum. This underinvestment is most severe if the quality leader captures the entire market in the pricing equilibrium. In this case the quality leader's profit depends on the marginal valuation its product, i.e. willingness to pay of the least quality-sensitive consumer. The firm's private value of information generation is thus lower than the social value, because the former depends on the marginal consumer's valuation, whereas the latter depends on the average consumer's valuation.[1]

Second, information acquisition generates a positive externality among firms, because drawing *any* signal increases a firm's expected profits, including signals about the opponent's product. Hence, even when price competition in the product market is perfect, firms can soften competition by cooperating in generating unbiased, publicly available information about product quality, for example, by introducing a classification system or an industry-wide competition. Such coordination decreases aggregate surplus whenever the level of information generation in the Nash equilibrium already exceeds the social optimum, which provides a novel perspective on the regulation of industry cooperation.

Finally, we endogenize the initial quality levels by allowing firms to costlessly degrade their quality before engaging in information generation. If firms do not expect any information generation, they will increase the expected quality distance in the product market by way of quality degradation. The possibility of generating public information mitigates this problem, because learning provides an alternative means to generate quality dispersion

[1] Interestingly, in the underinvestment case, consumers may benefit from generating additional information. Although we focus exclusively on the firms' problem, this implies that if consumers can overcome their collective action problem, then which side of the market generates information depends on the dead weight loss in the pricing equilibrium. For a recent contribution along these lines, see Terstiege and Wasser (2018).

in the market. Hence, when quality degradation is a concern, encouraging firms to cooperate in information generation may be socially desirable, as it prevents quality degradation.

While there is an active literature on information generation in monopoly settings, considerably less is known about settings with competition.[2] For instance, Ottaviani and Prat (2001) find that a monopolist always benefits from generating and disclosing signals that are affiliated to the valuation of the buyer. A number of authors studied the incentives to generate public information in auctions (see, for example, the seminal paper by Milgrom and Weber, 1982). Closer to our model, Ganuza and Penalva (2010) study an auction, in which one side of the market (the firm) generates information related to the *other* side of the market (the buyers' valuations). They argue that the amount of information generated by the firm will fall short of the social optimum, because information increases the dispersion in buyers' evaluations and information rents. Roesler and Szentes (2017) examine a related issue, identifying the optimal information environment from the buyers' point of the view. Our study focuses on the competitive effect generated by the presence of a second firm/seller, which affects the incentive to generate information even when the equilibrium outcome is a monopoly, because the identity of the monopolist may change with the arrival of new information.

The small literature that considers information generation in models with vertical differentiation tends to rely on very specific informational environments. For instance, Bouton and Kirchsteiger (2015) examine the role of reliable rankings of sellers and show that their presence can reduce consumers' welfare. Bergemann and Välimäki (2000) consider a dynamic setting, in which information is acquired exclusively through repeated purchases. They also find that information generation increases firms' market power and may reduce social welfare. We extend their setting by considering a more general form of information generation (any unbiased signal correlated with the

[2]There is, of course, the classical paradox that by construction fully revealing market equilibrium prices will not provide incentives for costly information acquisition (Grossman and Stiglitz, 1980), which has been resolved by allowing agents to take into account the effect of their actions on prices and beliefs of other agents (Milgrom, 1981; Verrecchia, 1982).

distribution of quality), characterize when information generation by firms falls short or exceeds the social optimum, examine the role of coordination in information generation, and consider the case of endogenous quality.

Related to our work is also Board (2009), who introduces information disclosure into a model of vertical competition. He shows that firms with private information about the quality of their product can use the choice of information disclosure to increase their vertical differentiation. Here we consider a similar environment, but focus on the choice of public information generation. A related literature has studied information disclosure and information generation in the context of horizontal competition. In particular, Anderson and Renault (2000, 2009) show that information generation may decrease consumers' welfare. Levin, Peck, and Ye (2009) consider a Hotelling model and argue that when firms operate as a cartel they may disclose more information than under a duopoly. These results are clearly related to ours. In both vertical and horizontal competition, increasing distance between quality levels via information generation increases firms market power. However, the welfare consequence of an increase in distance are very different in the vertical and horizontal case. This is particularly evident when the quality choice is endogenous, because increasing vertical product differentiation by quality degradation is unambiguously harmful for welfare.

Finally, a number of papers have studied models in which firms generate private information, that is, signals that are informative relative to each consumer's idiosyncratic preferences (for example, Lewis and Sappington, 1994, Moscarini and Ottaviani, 2001, Johnson and Myatt, 2006). We instead study public information, which is about the quality of the goods sold in the market, where "quality" refers to the attributes of a product that are valued by all consumers. Hence, all consumers prefer higher quality to lower quality but may solve the tradeoff between quality and price differently.[3]

The remainder of the paper is organized as follows. The next Section presents the model. Section 3 derives the equilibrium in the pricing game

[3]Because the taste for quality of each consumer is constant, but information generation affects the expected quality, we have that new information always shifts proportionally the consumer's willingness to pay.

for given expected qualities. Section 4 studies the incentives to generate information. In Section 5 we add a stage to the game in which each firm can degrade its product at no cost. The last section concludes. All mathematical derivations missing from the text are in Appendix.

2 Model

Our starting point is the canonical model of duopoly with vertically differentiated products (see Gabszewicz and Thisse, 1979, Shaked and Sutton, 1982, and Chapter 7 of Tirole, 1988's textbook). The market consists of 2 firms and a mass 1 of buyers. Each firm produces a good of quality $\tilde{s}_i \in [\underline{s}, \overline{s}]$ for $i \in \{1, 2\}$. A buyer's utility is given by

$$U = \begin{cases} \theta \tilde{s}_i - p_i & \text{if good } i \text{ is purchased} \\ 0 & \text{in case of no purchase,} \end{cases}$$

where p_i is the price of the good produced and θ is an i.i.d. taste parameter distributed over $[\underline{\theta}, \overline{\theta}]$, with $F(x) = \mathrm{pr}(\theta \leq x)$, continuous, differentiable, with continuous first derivative.

Each firm has zero marginal cost of production, so that profit is given by price times quantity sold.

Information and Learning

We depart from the canonical model by assuming that the quality levels $\tilde{s}_i$ are unknown to both buyers and firms, who have common ex-ante beliefs $q_i = E[\tilde{s}_i]$. Firms can generate more public information on the quality levels by acquiring informative signals on each product's quality. For example, the technical specifications of each product may be perfectly known by all market participants, which generates symmetric information and the common priors q_i. From the consumers' point of view, however, the consumption utility generated by purchasing the product $\tilde{s}_i$ may depend on subtle details unknown to either buyers or sellers.

Firm i can acquire information by paying a cost k and drawing a signal σ_i, which is informative with respect to $\tilde{s}_i$ and may be informative with respect to $\tilde{s}_{-i}$ as well.[4] We allow for any possible correlation between any σ_1 and σ_2. Information generated is public: all market participants receive the signal and update their belief about quality. Information generation is thus best understood as submitting the product to a public quality review process. A good example of such public reviews are classification systems, such as the ones for wine.[5] Notably, and for future reference, classification systems are often funded by industry bodies.[6]

Formally, the market's belief over the quality level of a firm i is given by a c.d.f. $G_i(s) : [\underline{s}, \overline{s}] \to [0, 1]$ that reflects the probability that the "true" quality $\tilde{s}$ is below a level s. Given $G_i(s)$, the expected quality level of the good sold by firm i is:

$$q_i = \int_{\underline{s}}^{\overline{s}} s \; dG_i(s).$$

Without loss of generality we assume $q_1 \geq q_2$. That is, before any additional information is generated the good sold by firm 1 has higher expected quality.

Call $\hat{\sigma}$ a specific realization of the vector of signals σ. Let $G_i(s|\hat{\sigma})$ denote the posterior belief distribution conditional on a signal $\hat{\sigma}$ and define the

[4] We abstract away from the choice of precision of the signal (as in the Bayesian persuasion literature, see in particular Gentzkow and Kamenica, 2016 who consider a competitive environment) as well as from the possibility of signal jamming. We will show that both firms' expected profits are increasing in the precision of both signals. Therefore, no firm will want to jam the other firm's signal, and, for given cost of drawing a signal, both firms will always choose the most precise signal available. Hence, our result will carry over to such a case. However, if signals of different precision had different cost, then firms would face a trade off in the choice of signal. This trade off would depend crucially on the rate at which the cost of each signal increases with its precision, and therefore prefer not to explore it.

[5] E.g., for Bordeaux wines the wine classification of 1855, and, more importantly, its more recent and regularly updated offshoot Cru Bourgeois, and similar systems in Burgundy, Champagne, Douro, and other regions. While observable variables such as soil quality of the vineyard and the weather of the vintage determine expected qualities, the true quality of a fine wine often only realizes after years of storage.

[6] For example, associations of vintners.

posterior expected quality as

$$\hat{q}_i = \int_{\underline{s}}^{\overline{s}} s \; dG_i(s|\hat{\sigma}).$$

We adopt the convention that $\sigma_i = \emptyset$, if firm i does not acquire information, so that $\hat{q}_i = q_i$ for $i \in \{1, 2\}$ if $\sigma = (\emptyset, \emptyset)$. We also write $\sigma = (\emptyset, \sigma_i)$ when firm $i \in \{1, 2\}$ acquires information, but not firm $-i$, and $\sigma = (\sigma_1, \sigma_2)$ when both firms acquire information. Note also that by iterating expectations $E[\hat{q}_i|\sigma] = q_i$ for any signal configuration σ. This means that ex ante, before any signals are drawn, the expected posterior quality is equal to the prior expected quality.

Timing

To summarize, the timing of the game is as follows.

1. $G_i(s)$ for $i \in \{1, 2\}$ is exogenously determined.[7]
2. Firms simultaneously decide whether to acquire information at cost k, yielding a vector of signals σ.
3. Realizations of signals $\hat{\sigma}$ are publicly revealed.
4. Firms announce prices simultaneously. Consumers decide if and from whom to buy and consume. Payoffs are realized.

Solution Concept

To derive the outcome of the game described above we employ a subgame perfect Nash equilibrium of signal generation choices σ_1 and σ_2 and price choices p_1 and p_2 depending on the signals.

Assumptions

We conclude the description of the model by introducing some restrictions on the distribution of the taste parameter θ. These restrictions guarantee the

[7] See Section 5 for the possibility that firms can affect expected quality q_i.

existence and uniqueness of a pure strategy Nash equilibrium in the pricing game (stage 4 in the above timeline), as we will show.

Assumption 1 (Full Support)**.** *Call $f(\theta)$ the p.d.f. of $F(\theta)$ and define $m \equiv \min_\theta\{f(\theta)\}$. We have:*

$$m > 0. \tag{A1}$$

Condition (A1) is equivalent to the distribution of the taste parameter θ having full support.

Assumption 2 (Log-concavity)**.** *The density $f(\theta)$ is log-concave.*

This assumption puts some very useful structure on the distribution of $F(\theta)$, ensuring that both $F(\theta)$ and $1 - F(\theta)$ are log-concave (see Prékopa, 1973 and Bagnoli and Bergstrom, 2005). This, in turns, implies that $F(\theta)/f(\theta)$ increases, $(1 - F(\theta))/f(\theta)$ decreases, and $(1 - 2F(\theta))/f(\theta)$ also decreases, all facts that we will use extensively in our derivations. As log-concavity is satisfied by a host of widely used distributions, this assumption comes with only a very modest loss of generality.

Finally, we impose the following parametric restriction:

Assumption 3 (Quality Sensitivity)**.**

$$\frac{\overline{s} - \underline{s}}{\overline{s} \cdot m} \leq \underline{\theta} \tag{A3}$$

Condition (A3) requires the least quality-sensitive consumer (of type $\underline{\theta}$) to be sufficiently responsive to quality. This condition is a generalization of the standard *covered market* condition.[8] As we will see, it guarantees that equilibrium prices are such that all consumers prefer to purchase from one of the firms to not purchasing. Note that we do not impose any constraint on

[8] For example, in Chapter 7 of Tirole (1988)'s textbook, the taste parameter is distributed uniformly with $\overline{\theta} - \underline{\theta} = 1$, and the model is solved assuming the covered market condition $\frac{|\hat{q}_1 - \hat{q}_2|}{\max\{\hat{q}_1, \hat{q}_2\}} \leq \underline{\theta}$. Condition (A3) is a generalization of this condition both because it applies to all possible distributions of the taste parameter, and because it applies to all possible quality levels (in Tirole, 1988 the quality levels are given and do not depend on the firm's choice of information acquisition.)

the upper bound of the taste parameter $\overline{\theta}$; therefore condition A3 can also be interpreted as a rescaling of the distribution of the taste parameter θ.[9]

3 The Pricing Game

Consider a given realization $\hat{\sigma}$ of the signal vector σ. If this realization is such that $\hat{q}_1 = \hat{q}_2$, then the two firms compete á la Bertrand and set equilibrium prices $p_1 = p_2 = 0$. All consumers purchase from one of the two firms, and are indifferent between purchasing from firm 1 or 2. If instead $\hat{q}_1 \neq \hat{q}_2$, there is scope for price differentiation between firms. Since information generation may reverse the initial quality ranking of firms, we will refer to the quality leader by L and the follower by F, so that $\hat{q}_L \equiv \max\{\hat{q}_1, \hat{q}_2\} > \hat{q}_F \equiv \min\{\hat{q}_1, \hat{q}_2\}$.

Denote a firm i's posted price by p_i. The demand for goods can be characterized by two thresholds. The first threshold X is given by the consumer who is indifferent between purchasing from either firm, if there is such a consumer, and by $\underline{\theta}$ $(\overline{\theta})$ if all consumers prefer L (F):

$$X \equiv \begin{cases} \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \geq \overline{\theta}. \end{cases}$$

The second threshold Y is given by the consumer who is indifferent between the lower quality firm F and not consuming, if there is such a consumer, and

[9] To the best of our knowledge, (1)-(3) are the weakest conditions existing in the literature guaranteeing existence, uniqueness and full analytical characterization of the pricing equilibrium with covered market. Papers solving the pricing equilibrium in the non covered-market case (Moorthy, 1988, Choi and Shin, 1992) or without imposing ex-ante whether the market will be covered (Wauthy, 1996) restrict their attention to uniform distribution of the taste parameter.

by $\underline{\theta}$ ($\overline{\theta}$) if all consumers prefer F (not to consume):

$$Y \equiv \begin{cases} \frac{p_F}{\hat{q}_F} & \text{if } \frac{p_F}{\hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \geq \overline{\theta}. \end{cases}$$

Noting that the quality leader can always out-price the follower these thresholds can be shown to have some useful properties.

Lemma 1. *In any pure strategy Nash equilibrium:*

- *A positive measure of consumers purchase from the quality leader: $Y \leq X < \overline{\theta}$.*

- *If not all consumers purchase from the quality leader ($X > \underline{\theta}$), then there is positive demand for the quality follower ($X > Y \geq \underline{\theta}$).*

Hence, in any pure strategy Nash equilibrium $\underline{\theta} \leq Y \leq X < \overline{\theta}$. Therefore the demand for good L is $1 - F(X)$ and the demand for good F is $F(X) - F(Y)$.[10] Profits are given by:

$$\pi_L(p_L, p_F) = p_L \cdot (1 - F(X)) \text{ and } \pi_F(p_L, p_F) = p_F \cdot (F(X) - F(Y)).$$

Given this, we can derive the two best responses:

Lemma 2. *The quality leader's best response is*

$$p_L(p_F) = \max\left\{\frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\},$$

which is a continuous function. The quality follower's best response is

$$p_F(p_L) = \begin{cases} [0, +\infty) & \textit{if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) \\ \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} & \textit{otherwise.} \end{cases}$$

which is a upper-hemicontinuous, compact valued, convex correspondence.

[10] From now on we focus exclusively on pure strategy equilibria, even when we do not explicitly mention this.

A Nash equilibrium is a tuple p^* such that $p_i(p_{-i}(p_i^*)) = p_i^*$ for $i = L, F$. Note that for the quality leader prices below $\underline{\theta}(\hat{q}_L - \hat{q}_F)$ are dominated, because the quality leader could increase its price and still capture the entire market, independently from the price set by the quality follower. Similarly, prices above $\overline{\theta}\hat{q}_L$ are also dominated for the quality leader, because they would lead to zero demand, and by Lemma 1 we know that there are always prices for which the quality leader faces positive demand. The equilibrium of the game can therefore be expressed as the fixed point of an upper-hemicontinuous, non empty, compact valued and convex correspondence $p_L(p_F(p))$ mapping a closed interval $[\underline{\theta}(\hat{q}_L - \hat{q}_F), \overline{\theta}\hat{q}_L]$ to itself. Then Kakutani's fixed point theorem applies and ensures existence of a pure strategy Nash equilibrium.

Depending on the distribution of the taste parameter θ, this equilibrium can be a monopoly or a duopoly. The following proposition gives sharp conditions on the type distribution for either case.

Proposition 1 (Market Equilibrium Outcome)**.**

(i) The pricing game has a unique pure strategy Nash equilibrium.

(ii) If $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$, then in equilibrium $X^* = Y^* = \underline{\theta}$, i.e. the quality leader supplies the entire market, and prices are $p_F^* = 0$ and $p_L^* = \underline{\theta}(\hat{q}_L - \hat{q}_F)$.

(iii) If instead $\frac{1}{f(\underline{\theta})} > \underline{\theta}$, then in equilibrium $Y^* = \underline{\theta}$ and $X^* > \underline{\theta}$ where

$$X^* = \frac{1 - 2F(X^*)}{f(X^*)} \tag{1}$$

Equilibrium prices are $p_L^* = (1 - F(X^*))/f(X^*)(\hat{q}_L - \hat{q}_F)$ and $p_F = F(X^*)/f(X^*)(\hat{q}_L - \hat{q}_F)$.

That is, $\underline{\theta} \cdot f(\underline{\theta})$ determines whether the market outcome will be a monopoly, with the quality leader supplying the whole market, or a duopoly, with the quality leader supplying consumers with $\theta \geq X^*$ and the quality follower supplying the remaining consumers. If $\underline{\theta} \cdot f(\underline{\theta}) \geq 1$ (monopoly) profits are given by

$$\pi_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = 0,$$

if instead $\underline{\theta} \cdot f(\underline{\theta}) < 1$ (duopoly) profits are given by

$$\pi_L = \frac{(1 - F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = \frac{(F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F).$$

Note also that the cutoff X^*, separating consumers buying from L from those buying from F, cannot be greater than median by construction. This leaves the quality leader always with at least half the market demand.

The important observation here is that while prices depend on posterior beliefs $\hat{q}_L$ and $\hat{q}_F$, and thus on the signal realizations $\hat{s}$, the demand faced by leader and follower does not. The demand faced by leader and follower depends only on the taste distribution $F(\theta)$, which determines whether there will be a monopoly or duopoly and the cutoff X^* in case of a duopoly. This in turn implies that the signal realizations, and therefore also the signal configurations, only affect the identity of quality leader and follower and the market prices that they can charge, which will very convenient in what follows.

To provide some illustration for Proposition 1 suppose that the taste distribution is uniform. Then the quality leader supplies the entire market if $\overline{\theta} \leq 2\underline{\theta}$, and there is a duopoly with $X^* = \frac{\underline{\theta}+\overline{\theta}}{3}$ otherwise. Which case will occur depends mainly on two intuitive effects. First, fixing either $\overline{\theta}$ or $\underline{\theta}$, as $\overline{\theta} - \underline{\theta}$ increases (and with it the variance of the distribution) the duopoly becomes more likely. This is because the quality leader will find it less profitable to attract the least quality sensitive consumers by lowering the price, thus leaving demand for the quality follower. By contrast, holding the range of the support $\overline{\theta} - \underline{\theta}$ constant, an equal increase in both $\overline{\theta}$ and $\underline{\theta}$ will make it more likely that the quality leader corners the market. This is because the incentive of the quality leader to sell to the least quality-sensitive consumer increases in her quality sensitivity.

4 Information generation

Equipped with the properties of the pricing equilibrium we are now in a position to examine the firms' choices of information acquisition. Depending on the properties of the type distribution, either the quality leader will corner the market (monopoly case) or both firms will supply some consumers. In the following we will consider both cases.

4.1 Case 1: Monopoly ($\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$)

We start by considering the case $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$, in which the quality leader covers the entire market. Since all consumers consume the good that has higher expected quality, the pricing equilibrium is efficient.

Social value of information generation. Given the expected qualities $\hat{q}_1$ and $\hat{q}_2$ the social welfare is given by:[11]

$$S(\hat{q}_1, \hat{q}_2) = \max\{\hat{q}_1, \hat{q}_2\}E[\theta]\,.$$

Suppose that no firm acquires information; then $\hat{q}_i = q_i$ for $i = 1, 2$ and (by assumption) firm 1 is the quality leader. The ex ante expected social welfare is then $S(q_1, q_2) = E[\theta]\, q_1 = E[\theta]\, E[\hat{q}_1|\sigma]$, where the last equality follows from the law of iterated expectation and holds for any σ. The social benefit of acquiring information is therefore given by the difference between expected social welfare given a chosen signal configuration σ and expected social welfare when no information is acquired:

$$\begin{aligned}
&E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) = \\
&\qquad E[\theta]\, E[\max\{\hat{q}_1, \hat{q}_2\}|\sigma] - E[\theta]\, E[\hat{q}_1|\sigma] = \\
&\qquad E[\theta]\{E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} + E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \\
&\qquad -E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} - E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\}\} = \\
&\qquad E[\theta]\, E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \equiv E[\theta]\, \Delta(\sigma, q_1, q_2),
\end{aligned}$$

where

$$\Delta(\sigma, q_1, q_2) \equiv \mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\}E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma], \tag{2}$$

[11]We assume throughout the paper that the cost of information generation k is a form of investment that, by itself, does not generate any social value.

is the *expected quality gain.* It measures the value of information generated by the signal configuration σ, i.e. the expected gain in quality in the event that the quality ranking changes and thus the good is supplied by a different firm, weighted by the probability that the quality ranking changes. If no signal is drawn, the quality ranking cannot change and $\Delta((\emptyset, \emptyset), q_1, q_2) = 0$. For any other configuration of σ the expected quality gain and thus the social benefit of learning are determined by the characteristics of signals and of the quality distribution. For instance, as the priors q_1 and q_2 get closer, given a signal configuration $\sigma \neq (\emptyset, \emptyset)$, also the probability that the quality ranking reverses and the expected quality gain in the event of a reversal will increase.

To summarize:because the two signals are informative with respect to the two qualities, then the law of iterated expectation applies and expected quality is independent from σ. In contrast, by a straightforward application of the Jensen's inequality, the expected maximum quality is a function of the signal configuration. More precisely, the above derivations show that, from a social point of view, information generation is beneficial if and only if there is a realization of the signal vector σ, such that the quality ranking reverses. If positive, the benefit of information generation increases in the probability of a ranking reversal and in the expected distance conditional on a ranking reversal.

For an intuition, suppose that a signal about the quality leader's quality is drawn. If the leader's quality is revealed to be better than expected, aggregate welfare will increase. If it is instead worse than expected, then aggregate welfare will decrease. The decrease in welfare is, however, bounded below by the quality of the ex-ante quality follower. This makes information generation about the quality leader socially beneficial in expectation. The case of the follower is analogous. This logic is remarkably general, extending for instance to the cases when each signal is informative of the other product as well, and when signals are correlated.

In our setting more information is better, in the sense that the expected quality gain is (weakly) monotone in the number of signals drawn:

$$\Delta((\emptyset, \emptyset), q_1, q_2) \leq \Delta((\emptyset, \sigma_i), q_1, q_2) \leq \Delta((\sigma_1, \sigma_2), q_1, q_2). \text{ for } i \in \{1, 2\} \quad (3)$$

This is because more information (in the form of two signals rather than one) increases the dispersion in the distribution of the posterior.[12] Note also that by definition the value of information generation $\Delta(\sigma, q_1, q_2)$ does not depend on which firm draws a signal, but only on the expected posterior quality distribution. Hence, if firms have access to the same signal technology so that the posterior quality distribution given σ_1 is the same as given σ_2, then the social value of information generation by firm 1 is identical to that by firm 2.

The socially optimal investment in information generation then solves

$$\max_{\sigma} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - \begin{cases} 2k \text{ if } \sigma = (\sigma_1, \sigma_2), \\ k \text{ if } \sigma = (\emptyset, \sigma_i) \text{ for } i \in \{1, 2\}, \\ 0 \text{ if } \ \sigma = (\emptyset, \emptyset). \end{cases}$$

Whether it is optimal to learn about only the quality leader, only the quality follower, or both will depend on the two expected qualities, on the two signals, and on the cost parameter k.

Private value of information generation. Given the outcome of the pricing game in Proposition 1 the firms' profits are

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \begin{cases} \underline{\theta}|\hat{q}_i - \hat{q}_{-i}| & \text{if } \hat{q}_i > \hat{q}_{-i} \\ 0 & \text{otherwise,} \end{cases}$$

which increase in the distance between quality levels, strictly so for the quality leader.

Lemma 3. *Consider any two signal configurations σ' and σ''. If there is a*

[12]To see this, suppose that the two signals are drawn sequentially starting from σ_1. The derivation above implies that for given $\hat{\sigma}_1$, drawing σ_2 is always weakly beneficial, strictly so if there are realizations of σ_2 for which the ranking given $\hat{\sigma}_1$ may change. Hence, in expectation (i.e., before drawing σ_1) information generation by both firms generates weakly greater social welfare (excluding the cost of experimentation) compared to information generation by only one firm.

monopoly (i.e., $\frac{1}{f(\underline{\theta})} > \underline{\theta}$) firm i's benefit from moving from σ' to σ'' is:

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \underline{\theta}\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right) \qquad (4)$$

Note that by (3) the benefit will be non-negative, if σ'' contains strictly more signals than under σ'.

Expression (4) can be used to compute the private benefit of information generation. If firm $-i$ is not drawing its signal, firm i's benefit from acquiring information is:

$$\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2)$$

The above expression is positive, if for some realization of σ_i the ranking between the two firms reverses. Intuitively, the quality follower benefits from generating information if and only if there is a chance of becoming the quality leader and thus make strictly positive profit. Perhaps somewhat surprisingly, the above expression implies that also the quality leader benefits from generating information if and only if there is a chance that the leader's product will be revealed to be worse than the competitor's. Similar to above, this is due to the fact that the gains and losses from generating information are asymmetric: the loss from a lower quality realization than expected is bounded below by a profit of zero, whereas the possible gains from a higher quality realization than expected are unbounded.

In the case that firm $-i$ acquires a signal, firm i's benefit from doing likewise is:

$$\underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_{-i}), q_1, q_2)\right).$$

This expression is always positive, and strictly so if drawing two signals rather than one either strictly increases the probability of a ranking reversal, or strictly increases the distance between quality levels conditional on a quality ranking reversal, or both.

We can now compare private and social benefits of information generation. Suppose the signal configuration σ' changes to σ'', containing one more signal than σ', which implies by (3) that $\Delta(\sigma'', q_1, q_2) \geq \Delta(\sigma', q_1, q_2)$. Then the difference between private and social benefits of information generation is given by

$$(\underline{\theta} - E[\theta])\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right) \leq 0,$$

which is always negative, and strictly so if $\Delta(\sigma'', q_1, q_2) > \Delta(\sigma', q_1, q_2)$ (that is, if σ'' is strictly more informative than σ'). The reason is that the private incentive to generate information is determined by the expected increase in the valuation of the least quality-sensitive consumer. Social benefits are instead determined by the expected increase in the average consumer's valuation. Note also that if the private benefit of information generation is lower than the social benefit, then information generation increases consumer surplus, because social welfare is the sum of profits and consumer surplus.

The following proposition states these observations.

Proposition 2. *Suppose* $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$*, i.e. there is a monopoly. Then each firm's private benefit from generating information is strictly lower than its social benefit. Information generation strictly increases consumer surplus.*

Proof. In the text. □

Finally, note that (4) implies that information generation of one firm generates an externality on the other firm. Suppose that firm i does not acquire a signal. Using (4) the effect on firm i's expected profit when firm $-i$ acquires a signal is:

$$\underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_i, q_{-i}).$$

Supposing instead that firm i acquires a signal as well, the effect of firm $-i$'s information generation on firm i's profits is:

$$\underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

The effect is positive in both cases, because of (3). That is, information generation of one firm generates a positive externality on the other firm's profit. This immediately implies the following lemma.

Lemma 4. *Firms' joint benefit from drawing a signal σ_i is greater than firm i's private benefit from drawing that signal.*

Proof. In the text. □

Subgame Perfect Nash Equilibrium A firm's optimal choice of whether to acquire a signal, and thus the outcome of the two stage game, will depend on whether the expected increase in profits computed above outweighs the investment cost k. That is, the subgame perfect Nash equilibrium of the information generation cum pricing game will depend on the quality distribution q_1 and q_2, the signal technology, and the investment cost k. We list the pure strategy equilibria below:

- If $k > \underline{\theta}\left(\Delta(\sigma_1, \sigma_2), q_1, q_2\right) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ then there is an equilibrium in which only firm $i \in \{1, 2\}$ generates information. There are two equilibria (each corresponding to a different firm generating information) whenever these inequalities hold both for $i = 1$ and $i = 2$.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$, but $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise, there is no information generation in equilibrium.

Therefore multiple equilibria are possible in two cases. First, when it is beneficial for each firm to generate information individually but not jointly, then there could be two equilibria depending on which firm generates information. Second, if the expected quality gain $\Delta(\sigma, q_1, q_2)$ increases in the number of signals drawn, then information acquisition choices will be strategic complements. An interesting case occurs when individual information generation is not profitable, but joint information generation is. This will be the case, for instance, when drawing one signal cannot perturb the posterior quality distribution sufficiently to reverse the quality ranking, but drawing two signals can (so that $\Delta((\emptyset, \sigma_1), q_1, q_2) = \Delta((\emptyset, \sigma_2), q_1, q_2) = 0$, but $\Delta((\sigma_1, \sigma_2), q_1, q_2) > 0$). If the cost k is sufficiently small, this case would produce a familiar coordination failure: an equilibrium without information generation, Pareto dominated by another equilibrium, in which both firms acquire information. In what follows, if there are multiple equilibria that can be Pareto ranked, we will always focus on the Pareto-preferred one.

Which of the different cases will emerge not only depends on the signal structure, but also on the distance in ex-ante expected qualities $|q_1 - q_2|$. If, for given signals, this distance is sufficiently small, information generation by at least one firm is more likely in equilibrium. For intermediate $|q_1 - q_2|$, there may be multiple equilibria, in which either both firms generate information or neither does, with the former equiibrium Pareto dominating the latter. If the distance is sufficiently large, neither firm will acquire any signal in equilibrium.

The equilibrium properties derived above and Proposition 2 imply the following proposition, derived in the appendix, stating that the market equilibrium will never generate more information than socially optimal.

Proposition 3. *Suppose $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$, i.e. there is a monopoly. Then the number of signals drawn in equilibrium is never higher than socially optimal. More precisely:*

(i) If for at least one $i \in \{1, 2\}$

$$E[\theta]\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k < E[\theta]\Delta((\emptyset, \sigma_i), q_1, q_2),$$

then it is socially optimal to draw one signal. If also for all $i \in \{1, 2\}$

$$k > \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

then in equilibrium no signals are drawn.

(ii) If

$$k \leq E[\theta] \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max\left\{\frac{1}{2}\Delta((\sigma_1, \sigma_2), q_1, q_2), \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\right),$$

then it is socially optimal to draw two signals. If also

$$k > \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\}\right)$$

then strictly less than two signals are drawn in equilibrium.

(iii) Otherwise the number of signals drawn in equilibrium is efficient. However, when it is socially optimal that only one firm draws a signal there may be multiple Nash equilibria, one with each firm drawing a signal, but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.

This statement implies that there cannot be overinvestment in information generation in equilibrium. However, firms are more likely to draw fewer signals in equilibrium than efficient, if the difference between private benefit (as measured by $\underline{\theta}$) and social benefit (as measured by $E(\theta)$) of information generation is large. We formalize this intuition in the following corollary.

Corollary 1. *Consider two distributions of the taste parameter $F(\theta)$ and $F'(\theta)$ with equal mean but different lower bounds $\underline{\theta} > \underline{\theta}'$ (possibly because one distribution is a mean-preserving spread of the other). The set of k for which there is an inefficient equilibrium under $F'(\theta)$ contains the set of k for which there is an inefficient equilibrium under $F(\theta)$.*

Coordination in information generation. When the firms can coordinate their information generation, they will choose a signal configuration that maximizes joint profits. By the previous derivations, the joint benefit of information generation by firm i is:

$$2\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

and the joint benefit of information generation by both firms is

$$2\underline{\theta}\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

Because information generation by one firm imposes a positive externality on the other firm, the firms' joint benefit from information generation is larger than each firm's individual benefit. Hence, firms may coordinate their choice of information generation via, for example, an industry body.[13] Therefore there are cost parameters k, for which no firm generates information in any equilibrium described above, but information generation by one or both firms will occur when firms jointly decide on information generation and share its cost. Similarly, for some level of k only one firm generates information in equilibrium, and both firms generate information when they can coordinate.

This increase in information generation will be socially beneficial, if the joint benefit of information generation is less than its social benefit, that is, when $2\underline{\theta} \leq E[\theta]$. In this case there is underinvestment in information generation both with and without coordination in information generation, but the underinvestment will be less severe when firms can coordinate.[14] These observations immediately yield the following corollary to our results above.

Corollary 2. *Firms that can coordinate their choice of information generation generate more information than is generated by individual choices in the Nash equilibrium. If $2\underline{\theta} \leq E[\theta]$, coordination increases social welfare and consumer surplus.*

4.2 Case 2: Duopoly ($\frac{1}{f(\underline{\theta})} > \underline{\theta}$)

Turn now to the case of a duopoly, i.e., a consumer type distribution that satisfies $\frac{1}{f(\underline{\theta})} > \underline{\theta}$ and thus implies both firms will sell to some consumers,

[13] As already noted, industry bodies are often responsible for creating and maintaining public information generation mechanisms such as classifications and competitions.

[14] When $2\underline{\theta} > E[\theta]$, firms coordination may lead to overinvestment in information generation, and may reduce social welfare. We discuss more in details this possibility in the next subsection.

jointly covering the market. While much of the analysis presented above carries over, contrary to the previous case here the pricing game leads to an inefficient outcome.

Social benefit of information generation. In the pricing equilibrium, the social welfare is given by:

$$\begin{aligned} S(\hat{q}_1, \hat{q}_2) &= \hat{q}_L \int_{X^*}^{\overline{\theta}} \theta dF(\theta) + \hat{q}_F \int_{\underline{\theta}}^{X^*} \theta dF(\theta) \\ &= \max\{\hat{q}_1, \hat{q}_2\}(1 - F(X^*))E[\theta|\theta > X^*] + \min\{\hat{q}_1, \hat{q}_2\}F(X^*)E[\theta|\theta < X^*] \\ &= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - |\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*]. \end{aligned} \tag{5}$$

The first part of this expression is the first-best social welfare, resulting from all consumers consuming the high quality good. The second part is the deadweight loss generated by positive demand for the lower quality good.

Information generation therefore has two competing effects on social welfare. Similar to the monopoly case above, drawing a signal increases the expected highest quality in the market, which increases social welfare. In contrast to the monopoly case above, information generation also increases the expected quality distance, which in turn increases the deadweight loss, too. The strength of this second effect depends on the market share of the quality follower (i.e., on $F(X^*)$) and on the average taste for quality of the consumers purchasing the low-quality good (i.e., $E[\theta|\theta < X^*]$). Both quantities strictly increase in X^*, which is therefore a sufficient statistics for the deadweight loss in the pricing equilibrium.

The expected change in social welfare due to information generation is given by:

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) = (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\, \Delta(\sigma, q_1, q_2) \geq 0. \tag{6}$$

The last inequality follows from the fact that, by the definition of X^*, the majority of consumers purchase the high quality good ($F(X^*) < 1/2$). It follows that:[15]

$$E[\theta] - 2F(X^*)E[\theta|\theta < X^*] > E[\theta] - E[\theta|\theta < X^*] > 0,$$

and the positive effect of information generation dominates. Therefore information generation always increases social welfare, strictly so when $\Delta(\sigma, q_1, q_2)$ is strictly positive.

Private benefit of information generation. To compare private and social returns, recall the firms' profits:

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = |\hat{q}_i - \hat{q}_2| \begin{cases} \frac{(1-F(X^*))^2}{f(X^*)} & \text{if } \hat{q}_i \geq \hat{q}_{-i} \\ \frac{F(X^*)^2}{f(X^*)} & \text{if } \hat{q}_i \leq \hat{q}_{-i}. \end{cases} \tag{7}$$

Both linearly increase in the distance between quality levels, but because $F(X^*) < \frac{1}{2}$ this increase is steeper for the quality leader.

Lemma 5. *Consider any two signal configuration σ' and σ''. If there is a duopoly (i.e., $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$), then firm i's benefit from moving from σ' to σ'' is:*

$$E\big[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''\big] - E\big[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'\big] = \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)\big(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\big) \tag{8}$$

Proof. Analogous to the one of Lemma 4 and therefore omitted. □

As in the case above the benefit will be non-negative, if σ'' contains strictly more signals than σ'.

We can use (8) to compute firms' private benefit of information acquisition. If firm $-i$ does not acquire information, the benefit of information generation for firm i is:

$$\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2).$$

[15]Recall also that in any Nash equilibrium of the pricing game the quality leader faces strictly positive demand and therefore $X^* < \overline{\theta}$.

Similarly, if firm $-i$ acquires information, the benefit of information generation for firm i is:

$$\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_{-i}), q_1, q_2))\,.$$

Comparing the private returns to information generation to the social ones derived above yields the following proposition.

Proposition 4. *Suppose $\frac{1}{f(\underline{\theta})} > \underline{\theta}$, i.e. there is a duopoly. If*

$$\frac{F(X^*)^2}{f(X^*)} + F(X^*)E[\theta|\theta < X^*] > \frac{1}{2}(E[\theta] - X^*), \tag{9}$$

then each firm's private benefit from generating information is strictly greater than its social benefit. If instead 9 is violated, then Proposition 2 applies: each firm's private benefit from generating information is strictly lower than its social benefit.

Proof. By comparing private benefit (equation 8) and social benefits (equation 6) of generating information. □

Note that the LHS of (9) strictly increases in the threshold consumer X^*, while the RHS of (9) is strictly decreasing in X^*. Hence, if X^* is sufficiently high (for example, close to the mean of the distribution of the taste parameter), then a firm's private benefit from acquiring a signal is higher than its social benefit. The reverse holds if X^* is sufficiently low (for example, close to the marginal consumer $\underline{\theta}$). Whether firms have an excessive incentive to invest in information generation thus depends on the characteristics of the type distribution.

For intuition, recall that the deadweight loss in the pricing equilibrium is $|\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*]$, and therefore strictly increases in X^*. Furthermore, X^* determines the sensitivity of the deadweight loss to the arrival of new information that increases the expected distance between qualities. Therefore the higher is X^*, the further from efficiency is the pricing equilibrium for a given quality distribution, and the more likely it is that the private value of information generation exceeds its social value. This reflects the fact that a higher deadweight loss is associated to a higher market share of the lower quality good. This in turn means that less consumers will benefit from a reversal of the quality ranking due to new information.

For illustration we turn again to the example of a uniform distribution, which permits easy computation. The private benefit of information acquisition exceeds the social benefit if $\overline{\theta} > \underline{\theta}(2+\sqrt{3})$ and falls short of it if $\overline{\theta} < \underline{\theta}(2+\sqrt{3})$. Again, fixing either $\overline{\theta}$ or $\underline{\theta}$, as the range of the support $\overline{\theta} - \underline{\theta}$ (and thus the variance of the distribution) increases, so does the deadweight loss in the pricing equilibrium. As a consequence the social value of information generation decreases, eventually dropping below its private value. On the other hand, if both $\overline{\theta}$ and $\underline{\theta}$ increase by the same amount, holding constant the range of the support $\overline{\theta} - \underline{\theta}$, the deadweight loss decreases. As a consequence, the social value of information generation increases, eventually becoming greater than its private value.

The subgame perfect equilibria of the information acquisition and pricing game can be derived analogously to the analysis above. Using the characterization of the equilibrium (see appendix) and Proposition 4 yields the following analogue of Proposition 3.

Proposition 5. *Suppose* $\frac{1}{f(\underline{\theta})} > \underline{\theta}$, *i.e. there is a duopoly, and that Condition (9) holds. Then the number of signals drawn in a Pareto-dominant Nash equilibrium is never lower than socially optimal. More precisely:*

(i) If

$$k > (E[\theta] - 2F(X^*)E[\theta|\theta < X^*]) \\ \max\left\{\Delta((\emptyset,\sigma_1),q_1,q_2), \Delta((\emptyset,\sigma_2),q_1,q_2), \frac{1}{2}\Delta((\sigma_1,\sigma_2),q_1,q_2)\right\}.$$

then it is socially optimal to have no information generation. If also either

$$k \leq \left(X^* + 2\cdot\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset,\sigma_i),q_1,q_2) \text{ for at least one } i \in \{1,2\},$$

or

$$k \leq \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) \quad \forall i \in \{1, 2\}.$$

then the number of signals drawn in a Pareto-dominant Nash equilibrium is strictly positive.

(ii) If for at least one $i \in \{1, 2\}$

$$(E[\theta] - 2F(X^*)E[\theta|\theta < X^*]) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) < k < (E[\theta] - 2F(X^*)E[\theta|\theta < X^*]) \Delta((\emptyset, \sigma_i), q_1, q_2),$$

then it is socially optimal for one firm to generate information. If also

$$k \leq \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) \quad \forall i \in \{1, 2\}.$$

then in a Pareto-dominant Nash equilibrium both firms draw a signal each.

(iii) Otherwise the number of signals drawn in equilibrium is efficient. However, when it is socially optimal that only one firm draws a signal, there may be multiple Nash equilibria, one with each firm drawing a signal but not the other. One of these equilibria is inefficient because, the firm with the less informative signal generates information.

That is, if Condition (9) holds, the market equilibrium level of information generation in a duopoly will never fall short of the social optimum and there may be over-investment in information generation. If instead Condition (9) does not hold, the private benefit from information generation falls short of the social benefit, then Proposition 3 applies:[16] the number of signals drawn in equilibrium is never higher than socially optimum and there may be under-investment in information generation. In both cases it is also possible that,

[16]That is, a version of Proposition (9) which accounts for the different expressions for private and social benefits of information generation (which, in Proposition (9) are $\underline{\theta}$ and $E(\theta)$ and should instead be $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right)$ and $(E[\theta] - 2F(X^*)E[\theta|\theta < X^*])$, respectively).

in equilibrium, the "wrong" firm invests in information generation.

Note that, again, if Condition (9) holds in equilibrium firms are more likely to draw more signals than efficient when the difference between private benefit (as measured by $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)$) and social benefit (as measured by $(E[\theta] - 2F(X^*)E[\theta|\theta < X^*])$) of information generation is large. By log concavity, for a given $E[\theta]$ this difference is strictly increasing in X^*, which implies the following corollary.

Corollary 3. *Consider two distributions of the taste parameter $F(\theta)$ and $F'(\theta)$ having equal mean but different $X^{*\prime} > X^*$. If Condition (9) holds at both $F(\theta)$ and $F'(\theta)$, then the set of k for which there is an inefficient equilibrium under $F'(\theta)$ contains the set of k for which there is an inefficient equilibrium under $F(\theta)$. If Condition (9) is violated at both $F(\theta)$ and $F'(\theta)$, then the set of k for which there is an inefficient equilibrium under $F'(\theta)$ is contained in the set of k for which there is an inefficient equilibrium under $F(\theta)$.*

Recall that the deadweight loss in the pricing equilibrium is also strictly increasing in X^*. The above corollary implies that, starting from no deadweight loss (so that (9) is violated), as the deadweight loss increases the set of k for which there is under investment in information generation progressively shrink to zero. As the deadweight loss increases further, condition (9) holds and the set of k for which there over investment in information generation expands. The relation between inefficiencies in the pricing stage and inefficiencies in the information-generation stage is therefore non-monotonic.

Coordination in information generation. As above information generation by one firm imposes a positive externality on the other firm. Hence, the firms' joint benefit of information generation exceeds each individual firm's private benefit.

To show this, suppose that firm i does not acquire a signal. By (8) the effect of $-i$'s information generation on firm i's expected profit is:

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|(\emptyset, \sigma_i)] - \pi(q_i, q_{-i}) = \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta((\emptyset, \sigma_{-i}), q_1, q_2) \geq 0,$$

with a strict inequality if the expected quality gain is strictly positive. If instead firm i has decided to acquire a signal as well, firm $-i$'s information generation changes i's profits as follows:

$$
\begin{aligned}
& E[\pi_i(\hat{q}_i, \hat{q}_{-i})|(\sigma_1, \sigma_2)] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|(\emptyset, \sigma_i)] \\
&= \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) \geq 0,
\end{aligned}
$$

where, again, the inequality follows from (3).

The fact that each firm's information generation generates a positive externality on the other firm's profit has an interesting implication for consumer's welfare. If condition (9) holds, the number of signals drawn in a Nash equilibrium is greater than socially optimal. Allowing for collusion in information generation can only increase the number of signals drawn, which will thus increase firms' joint profits, but decrease social surplus. This observation implies the following corollary.

Corollary 4. *If (9) holds, then coordination in information generation decreases aggregate welfare and consumer surplus.*

If instead (9) does not hold, then similarly to the monopoly case examined above coordination in information generation may increase welfare .

5 Extension: endogenous quality

One possibility that we have so far ignored is that firms may intentionally degrade their products to increase the distance between their qualities. For example, in his textbook Tirole (1988) shows that, if quality degradation is costless, then the quality follower should always degrade its quality as much as possible in order to achieve maximum distance from the quality leader. Quality degradation is observed in some instances,[17] but is far from ubiquitous. In this section we argue that information generation may act

[17] For example, several producers of electronic devices are known to intentionally reduce the performance and functionality of their products; a well known example is the case of IBM printers.

as a strategic substitute to quality degradation, and therefore explain why quality degradation is rarely observed. This also implies that, when quality is endogenous, information generation has an additional social benefit because it may prevent harmful quality degradation.

Denote by $q_i^0 \in [\underline{s}, \overline{s}]$ firm i's initial quality with the convention that $q_1^0 > q_2^0$. Before the market opens, both firms simultaneously can decrease their expected quality at zero cost to any $q_i \in [\underline{s}, q_i^0]$.[18] Quality degradation is publicly observable. Recall that in the duopoly case of firms' profits increase in the distance between their expected quality levels. It follows that, absent information generation, in the pure strategy Nash equilibrium of a quality degradation game the quality leader will maintain the default quality $q_1 = q_1^0$ and the quality follower will degrade as much as possible to $q_2 = \underline{s}$.[19]

Turn now to a quality degradation and information generation game: after deciding whether to degrade its quality each firm can acquire information at a cost k. Introducing information generation may affect the choice quality degradation, because information generation provides an alternative means to increase the quality distance between firms. However, in contrast to degradation, information generation allows for upward revisions of the expected quality as well as downward revisions, increasing the expected highest quality and thus aggregate surplus.

Given that at least one firm generates information, the quality follower's profit can be written as:

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] = \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) \Delta(\sigma, q_1, q_2) + \pi_2(q_1, q_2). \tag{10}$$

[18]More precisely, firm i can shift downward the distribution of belief about its quality $F_i(s)$ so to achieve any expected quality $q_i \in [\underline{s}, q_i^0]$. As discussed above the "consumption utility" generated by consuming a product $\tilde{s}_i$ is unknown, but the technical specifications of the product are publicly known and determine the expectation of $\tilde{s}_i$. With this interpretation in mind, quality degradation can be achieved by designing a product with worse technical specifications.

[19]The pure-strategy Nash equilibrium in which the quality follower degrades always exists. A second pure-strategy Nash equilibrium, in which the quality leader fully degrades its quality, but the quality follower does not, exists for some q_1^0, q_2^0. In case both equilibria exist, they can be ranked in terms of efficiency, because the welfare loss is smaller when the quality follower degrades than when the quality leader degrades. For ease of exposition, we only discuss the Nash equilibrium in which the quality follower degrades.

Note that $\pi_2(q_1, q_2)$ decreases in q_2. However, $\Delta(\sigma, q_1, q_2)$ increases in q_2, and strictly so if $\Delta(\sigma, q_1, q_2) > 0$. Hence, if $\Delta(\sigma, q_1, q_2) = 0$ and information generation has no value, the above result carries over and the quality follower is better off by degrading as much as possible to maximize the distance to the quality leader. When $\Delta(\sigma, q_1, q_2) > 0$, however, it is possible that $E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma]$ increases in q_2, and hence that there is no incentive to degrade quality. That is, quality degradation and information generation can be alternative ways to achieve vertical differentiation.

In the following we provide a sufficient condition for this case to occur. Note that

$$\pi_2(q_1, q_2) = \frac{F(X^*)^2}{f(X^*)}(q_1^0 - q_2),$$

is arbitrarily close to zero whenever X^* is close to $\underline{\theta}$, because in this case the demand for the good sold by the quality follower is arbitrarily small. It is also arbitrarily close to zero if q_1^0 is close to $\underline{s}$, because the maximum distance that can be achieved between quality leader and follower is also arbitrarily small. In either of these cases, we have that

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] \approx \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2),$$

so that the quality follower's profit is strictly positive and strictly increases in q_2, if $\Delta(\sigma, q_1^0, q_2^0)$ is strictly positive, that is, if q_1^0 and q_2^0 are sufficiently close or if σ is sufficiently informative (in the sense of dispersion in the posterior beliefs). This observation implies the following lemma.

Lemma 6. *For any X^*, there are q_1^0, q_2^0 such that the quality follower will fully degrade quality if no information is generated, but neither firm will degrade quality if information is generated by at least one firm.*

Proof. In the text. □

The lemma states that information generation can prevent harmful quality degradation. Indeed, a sufficiently low cost k will guarantee some information generation in equilibrium, which implies the following corollary.

Corollary 5. *There are q_1^0, q_2^0 and k such that information generation and no quality degradation constitute a subgame perfect Nash equilibrium of the quality degradation and information generation game.*

Note that, if the case described in Lemma 6 and the following corollary do not apply, the quality follower may partially degrade, even though information is generated in equilibrium. For example, if the signals σ_1 and σ_2 are discrete, then the probability of a quality ranking reversal may be discontinuous in the amount of quality degradation by the quality follower. That is, this probability may be very small when the quality follower degrades by a small amount, but jump discontinuously if the amount of quality degradation passes a given threshold. If, at the same time, the benefit of increasing vertical distance is large, the quality follower may prefer to partially degrade quality. We will not consider this possibility here.

Turning to social welfare, supposing that information generation prevents quality degradation, as in the case described in Lemma 6, the associated social benefit is:

$$
\begin{aligned}
&E[S(\hat{q}_1, \hat{q}_2)] - S(q_1^0, \underline{s}) = \\
&E[S(\hat{q}_1, \hat{q}_2)] - S(q_1^0, q_2^0) + S(q_1^0, q_2^0) - S(q_1^0, \underline{s}) = \\
&(E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\, \Delta((\emptyset, \sigma_i), q_1^0, q_2^0) + (q_2^0 - \underline{s})F(X^*)E[\theta|\theta < X^*].
\end{aligned} \tag{11}
$$

The first term of this expression is the benefit of information generation given initial quality levels as in (6). The second term stems from (5) and is the benefit from preventing quality degradation. It increases in $q_2^0 - \underline{s}$ (the amount of quality degradation prevented by generating information), in the mass of consumers purchasing from the quality follower, and in the average valuation of these consumers. The following proposition summarizes these observations.

Proposition 6. *There are q_1^0, q_2^0 and k such that the social benefit of information generation with endogenous quality is strictly greater than the one with exogenous qualities.*

Proof. Directly from Lemma 6, expression (11) and the following discussion. □

The fact that information generation can have an additional social benefit when quality is endogenous rather than exogenous implies that Proposition 5 may not apply. That is, there are cases such that, when quality is exogenous, the efficient number of signals is zero but in equilibrium at least one firm generates information. Under endogenous quality choice the fact that a firm is expected to generate information prevents quality degradation. If the social benefit of preventing quality degradation (given by the second part of 11) is larger than the net social cost of drawing an additional signal (given by the second part of 11 minus k), then having one firm generating information is the socially optimal outcome with endogenous quality levels. Similarly, with exogenous quality levels there are situations in which the efficient number of signals is zero, which is also the equilibrium outcome. With endogenous quality levels, however, the absence of information generation leads to quality degradation and may, therefore, be inefficient. The following corollary summarizes these observations.

Corollary 6. *Suppose the case described in Lemma 6 holds. There are cases in which there is over-investment in information generation with exogenous quality, but the efficient level of information generation when quality levels are endogenous. Similarly, there are cases in which there is the efficient level of information generation with exogenous quality, but under-investment in information generation when quality levels are endogenous.*

6 Conclusion

We consider a standard duopoly with vertically differentiated products, and study firms' incentives to generate information. Our main result is that firms will under- or overinvest in information generation, depending on the inefficiencies in the pricing equilibrium. When, for a given quality distribution, the deadweight loss in the pricing equilibrium is low, firms will under-provide information. Conversely, when the deadweight loss is large in equilibrium, firms will over-provide information. We also show that information generation has a positive externality on the other firm's profit and thus firms benefit from coordinating their information acquisition activities. Finally, we introduce the possibility of quality degradation and show that quality degradation and information generation are substitutes for increasing vertical product differentiation. Therefore the possibility of information generation may reduce harmful quality degradation.

This last result implies that there are situations in which information generation should be discouraged if quality levels are exogenous—possibly via a tax—but information generation should be encouraged if quality levels are endogenous—possibly via a subsidy. This insight carries over to a more concrete policy application, i.e. the extent to which cooperation and coordination of competing firms will result in overinvestment in information generation. Our analysis shows that there are situations in which coordination in information generation should be prevented if quality levels are exogenous, but should be allowed or even encouraged if quality levels are endogenous. This, however, implies that the optimal policy may be time inconsistent, because the policymaker may revise its policy after the quality levels are set, just before the choice of information generation. We leave the exploration of this issue to future work.

Appendix

Proof of Lemma 1

Note first that $X = Y = \overline{\theta}$ quickly leads to a contradiction: if both prices are so high that no consumers purchase, then each one of the firms will earn strictly positive payoff by deviating to a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Suppose that $X = \overline{\theta}$, i.e. the quality leader faces zero demand. Then, by the argument above, $Y < \overline{\theta}$ and the quality follower faces positive demand. This cannot be an equilibrium because the quality leader can set its price equal that of the quality follower, generate positive demand and earn positive profits.

Finally, suppose that $\underline{\theta} < X < \overline{\theta}$, i.e. the quality leader faces positive demand, but does not capture the entire market. Then $Y < X$. To see this suppose the contrary, i.e. $Y = X$. This cannot be an equilibrium because the quality follower will earn strictly positive payoff by setting a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Proof of Lemma 2

The best responses are:

$$\begin{aligned} p_L(p_F) &= \text{argmax}_{p_L} \{\pi_L(p_L, p_F)\} \\ p_F(p_L) &= \text{argmax}_{p_F} \{\pi_F(p_L, p_F)\} . \end{aligned}$$

Consider first the quality leader's problem. For given p_F the leader can out-price the follower and set $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, so that $X = \underline{\theta}$. In this case the leader serves the entire market and its profit equals p_L. Hence, conditional on $X = \underline{\theta}$ the quality leader maximizes profits by setting $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$. If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, the leader serves only a fraction of the total market, and $X > Y \geq \underline{\theta}$. Using the definition of X the quality leader's problem becomes:

$$\max_{p_L \geq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F} \left\{ p_L \left(1 - F\left(\frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \right) \right) \right\} .$$

The first derivative of the objective function is

$$\left(\frac{1 - F(X)}{f(X)} - \frac{p_L}{\hat{q}_L - \hat{q}_F} \right) f(X),$$

and equals zero at $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which is unique due to log-concavity. Log-concavity also implies that the second derivative of the objective function is negative at $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. The quality leader's objective function therefore strictly increases for $p_L < \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and strictly decreases for $p_L > \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

We now turn to the quality follower F's best response. Suppose the quality leader chooses a price $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F)$. In this case for any p_F the quality leader covers the entire market and the quality follower's profits are zero for any p_L. Then the quality follower's best response is

$$p_F(p_L) = [0, \infty) \text{ if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F).$$

If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F)$, there are $p_L > 0$, such that both the demand faced by and the profit of the quality follower are positive. In this case the follower's profit function has a kink at price $p_F = \underline{\theta}\hat{q}_F$. Although the profit function is not differentiable at $p_F = \underline{\theta}\hat{q}_F$, it is well-behaved above and below, which allows us to characterize the follower's best response.

If $p_F \leq \underline{\theta}\hat{q}_F$, then all consumers purchase one of the goods and $Y = \underline{\theta}$, so that a change in p_F only affects X. Conditional on $Y = \underline{\theta}$, the follower's profit function is

$$\max_{p_F \leq \underline{\theta}\hat{q}_F} \{p_F F(X)\}.$$

The objective function's first derivative is

$$\left(\frac{F(X)}{f(X)} - \frac{p_F}{\hat{q}_L - \hat{q}_F}\right) f(X).$$

The first derivative equals zero at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which is unique by log-concavity. Again, conditional on $Y = \underline{\theta}$, the follower's profit function is strictly concave at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which in turns imply that profits conditional on $Y = \underline{\theta}$ are first increasing then decreasing in p_F, reaching a maximum at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

If instead $p_F > \underline{\theta}\hat{q}_F$ some consumers will not purchase, so that a change in p_F will affect both X and Y. Conditional on $\underline{\theta} \leq Y < X$, the follower's profit function is now

$$\max_{p_F \geq \underline{\theta}\hat{q}_F} \{p_F(F(X) - F(Y))\}.$$

The objective function's first derivative is

$$F(X) - F(Y) - p_F \left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Now Condition (A3) becomes useful: it implies that the above expression is always negative, which implies that the quality follower always sets a price so that $Y = \underline{\theta}$ and the market is covered.

To see why, recall that the first order condition for the case $Y > \underline{\theta}$ is

$$F(X) - F(Y) - p_F \left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Because $F(X) - F(Y) \leq 1$, $p_F > \underline{\theta}\hat{q}_F$ (whenever $Y > \underline{\theta}$), $\frac{\hat{q}_F}{\hat{q}_L} \geq \frac{\underline{s}}{\overline{s}}$, and $f(X), f(Y) \geq m$, the above expression is always smaller than

$$1 - m\underline{\theta} \left(1 - \frac{\underline{s}}{\overline{s}} \right)^{-1},$$

which is negative under (A3). Hence the first order condition for the case $Y > \underline{\theta}$ is always negative, and the quality follower is always better off by setting p_F such that $Y = \underline{\theta}$.

Proof of Proposition 1

As a preliminary step, note that (A3) is equivalent to

$$\frac{1}{m\underline{\theta}} - 1 \leq \left(\frac{\overline{s}}{\overline{s} - \underline{s}} \right) - 1$$

$$\left(\frac{1}{m\underline{\theta}} - 1 \right) \left(\frac{\overline{s}}{\underline{s}} - 1 \right) \leq 1 \tag{12}$$

We previously derived the quality leader's best reply

$$p_L(p_F) = \max \left\{ \frac{1 - F(X)}{f(X)} (\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F \right\}$$

By log-concavity $\frac{1-F(X)}{f(X)}$ is decreasing and therefore is at most $\frac{1}{f(\underline{\theta})}$. Hence, whenever $\frac{1}{f(\underline{\theta})} \leq \underline{\theta}$ the quality leader's best reply is $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$ and he captures the entire market. If $p_F > 0$ this cannot be an equilibrium because the quality follower should lower its price and earn positive profits. If instead $p_F = 0$ and $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F)$ then no firm can make a profitable deviation, and these prices constitute a Nash equilibrium.

Suppose instead $\frac{1}{f(\underline{\theta})} > \underline{\theta}$ from now on. The observations made in the text above imply that in this case the quality leader's best reply to $p_F = 0$ is $p_L = \frac{1-F(p_L/(\hat{q}_L - \hat{q}_F))}{f(p_L/(\hat{q}_L - \hat{q}_F))}(\hat{q}_L - \hat{q}_F)$. Hence, by Lemma 1 the Nash equilibrium will necessarily have $X > \underline{\theta}$ and $p_F = \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} > 0$. Therefore there are two possible cases, depending on whether the quality follower's best response is a corner solution ($p_F = \underline{\theta}\hat{q}_F$) or an interior solution ($p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$).

Suppose first that the quality follower's best response has an interior solution:

$$\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F) \leq \underline{\theta}\hat{q}_F, \tag{13}$$

so that $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. In this case, by definition of X, the equilibrium cutoff X solves

$$X = \frac{1 - 2F(X)}{f(X)}. \tag{14}$$

This equation has a unique solution because, by log concavity, its RHS is decreasing in X and we have assumed $\underline{\theta} < \frac{1}{f(\underline{\theta})}$. This constitutes a Nash equilibrium if indeed the solution X of equation (14) satisfies condition (13). Condition (13) can be rewritten as

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L}{\hat{q}_F} - 1\right) \leq 1.$$

Note that $\frac{\hat{q}_L}{\hat{q}_F}$ is at most $\frac{\overline{s}}{\underline{s}}$, and that by (14) $\frac{F(X)}{f(X)} = \frac{1}{2}\left(\frac{1}{f(X)} - X\right)$, which is at most $\frac{1}{2}\left(\frac{1}{m} - \underline{\theta}\right)$. Therefore,

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L}{\hat{q}_F} - 1\right) < \frac{1}{2}\left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) < 1,$$

where the last inequality follows by (12). Hence, (13) holds and thus $p_F =$

$\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, with X defined implicitly by (14) is a Nash equilibrium.

To conclude the proof, we show that there is no equilibrium in which $\frac{1}{f(\underline{\theta})} > \underline{\theta}$, and hence the quality leader's best response has an interior solution:

$$p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F),$$

but at the same time (13) is violated, and hence the quality follower's best response has a corner solution:

$$p_F = \underline{\theta}\hat{q}_F.$$

If such equilibrium exists, then by definition

$$X = \frac{1-F(X)}{f(X)} - \underline{\theta}\left(\frac{\hat{q}_L}{\hat{q}_F} - 1\right)^{-1} \tag{15}$$

This is consistent with a Nash equilibrium if indeed for this X (13) is violated.

Note that by (15) $\frac{F(X)}{f(X)}$ is smaller than $\frac{1}{f(X)} - X$ which, in turn, is smaller than $\frac{1}{m} - \underline{\theta}$. Also, $\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}$ must be smaller than $\left(\frac{\overline{s}}{\underline{s}} - 1\right)$. It follows that

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}\right) \leq \left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1,$$

where the last inequality follows by (12). Hence, (13) must hold and there cannot be a Nash equilibrium with $p_F = \underline{\theta}\hat{q}_F$.

Proof of Lemma 3

Suppose $\sigma' = (\emptyset, \emptyset)$, that is, we are compare signal configuration σ'' to no information. Since firm 1 is the quality leader ex ante, if no information is generated then $\pi_1(q_1, q_2) = \underline{\theta}\,[q_1 - q_2]$ and $\pi_2(q_1, q_2) = 0$. Therefore firm i's benefit of signal configuration σ'' relative to no information is:

$$E\big[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma''\big]-\pi_i(q_i,q_{-i}) =$$

$$E\big[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma''\big]-\underline{\theta}\begin{cases}E\left[\hat{q}_1-\hat{q}_2|\sigma''\right] & \text{if } i=1\\ 0 & \text{if } i=2\end{cases}$$

$$=\underline{\theta}\text{pr}\{\hat{q}_i\geq\hat{q}_{-i}|\sigma''\}E\big[\hat{q}_i-\hat{q}_{-i}|\hat{q}_i\geq\hat{q}_{-i},\sigma''\big]-$$

$$\underline{\theta}\cdot\begin{cases}\text{pr}\{\hat{q}_2\geq\hat{q}_1|\sigma''\}E[\hat{q}_1-\hat{q}_2|\hat{q}_2\geq\hat{q}_1,\sigma'']+\text{pr}\{\hat{q}_1\geq\hat{q}_2|\sigma''\}E[\hat{q}_1-\hat{q}_2|\hat{q}_1\geq\hat{q}_2,\sigma''] & \text{if } i=1\\ 0 & \text{if } i=2\end{cases}$$

$$=\underline{\theta}\text{pr}\{\hat{q}_2\geq\hat{q}_1|\sigma''\}E\big[\hat{q}_2-\hat{q}_1|\hat{q}_2\geq\hat{q}_1,\sigma''\big]=\underline{\theta}\Delta(\sigma'',q_1,q_2),$$

where, again, we used the fact that by the law of iterated expectations $\pi_1(q_1,q_2)=\underline{\theta}(q_1-q_2)=\underline{\theta}E\left[\hat{q}_1-\hat{q}_2|\sigma''\right]$.

The above derivation also implies that the benefit of going from no information to signal configuration σ' is

$$E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma']-\pi_i(q_i,q_{-i})=\underline{\theta}\Delta(\sigma',q_1,q_2).$$

We therefore have

$$E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma'']-E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma']=$$

$$(E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma'']-\pi_i(q_i,q_{-i}))-(E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma']-\pi_i(q_i,q_{-i}))=\underline{\theta}\left(\Delta(\sigma'',q_1,q_2)-\Delta(\sigma',q_1,q_2)\right).$$

Proof of Proposition 3

We distinguish three cases

1. It is socially optimal to generate no information, that is

$$2k>E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2) \text{ and } k>E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2)\quad i\in\{1,2\}$$

   By Proposition 2 each firm's best reply to the other firm not generating information is to not generate information either. Likewise, each firm i's best reply to the other firm $-i$ generating information is not to generate information, if

$$k \geq \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_{-i}), q_1, q_2) \right),$$

which is always true, because

$$\begin{aligned} \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2) \right) &\leq E[\theta] \Delta((\sigma_1, \sigma_2), q_1, q_2) - \underline{\theta} \Delta((\emptyset, \sigma_i), q_1, q_2) \\ &\leq 2k - \underline{\theta} \Delta((\emptyset, \sigma_i), q_1, q_2) \leq k \end{aligned}$$

Hence, in the case when no information generation is socially optimal there is a unique Nash equilibrium, in which neither firm generates any information.

2. It is socially optimal for firm i to generate information, but not firm $-i$, that is

$$\begin{aligned} E[\theta] \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2) \right) &< k < E[\theta] \Delta((\emptyset, \sigma_i), q_1, q_2), \\ E[\theta] \Delta((\emptyset, \sigma_{-i}), q_1, q_2) &\leq E[\theta] \Delta((\emptyset, \sigma_i), q_1, q_2). \end{aligned} \tag{16}$$

The first inequality immediately implies

$$k > \underline{\theta} \left[\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2) \right].$$

This means that if firm i generates information, then firm $-i$'s best reply is to not generate information. Hence, there is no Nash equilibrium, in which both firms generate information.

Suppose that

$$k > \underline{\theta} \Delta((\emptyset, \sigma_i), q_1, q_2). \tag{17}$$

Then neither firm finds it profitable to generate information if the other firm does not. Hence, in the unique Nash equilibrium there is no information generation.

If instead

$$\underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2) < k \leq \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

then there is a unique equilibrium in which firm i generates information. Finally, if

$$k \leq \underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2),$$

then there are multiple equilibria, in which each firm may generate information, while the other one does not. In one of these equilibria firm $-i$ generates information, but not firm i. This is inefficient, because by assumption $\Delta((\emptyset, \sigma_i), q_1, q_2) < \Delta((\emptyset, \sigma_{-i}), q_1, q_2)$, i.e. firm i's signal generates more information (as measured by the dispersion of the posteriors) and higher social welfare than firm $-i$'s signal.

3. It is socially optimal for both firms to generate information, that is

$$\begin{aligned}
&2k \leq E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) \\
&E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k \geq E[\theta]\Delta((\emptyset, \sigma_1), q_1, q_2) - k \\
&E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k \geq E[\theta]\Delta((\emptyset, \sigma_2), q_1, q_2) - k
\end{aligned}$$

for $i \in \{1, 2\}$. That is, the net social benefit of drawing both signals is positive, and exceeds the net social benefit of drawing either individual signal. The above inequalities can be rewritten as

$$\begin{aligned}
k \leq E[\theta]\, &(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \\
&\max\left\{\frac{1}{2}\Delta((\sigma_1, \sigma_2), q_1, q_2), \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\Bigg)
\end{aligned}$$

A necessary condition for both firms to generate information in a Nash equilibrium (including the case of multiple equilibria) is

$$k < \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

for both firms $i = 1, 2$, or

$$k \leq \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max \left\{ \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2) \right\} \right)$$

Therefore, if

$$k > \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max \left\{ \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2) \right\} \right) \tag{18}$$

and

$$\begin{aligned} k \leq E[\theta] \, (& \Delta((\sigma_1, \sigma_2), q_1, q_2) - \\ & \max \left\{ \frac{1}{2} \Delta((\sigma_1, \sigma_2), q_1, q_2), \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2) \right\} \Bigg) \end{aligned} \tag{19}$$

then the number of signals drawn in a Nash equilibrium is stricly less than in the social optimum. Otherwise, there will be a (possibly unique) Nash equilibrium that is efficient.

We therefore established that the number of signals drawn in equilibrium is always smaller than the socially optimal number of signals, strictly so if either both conditions (16) and (17) hold, or both conditions (18) and (19) hold. Note also that, in both cases, the set of such k for which fewer signals than optimal are drawn expands with $E[\theta] - \underline{\theta}$ and with the first difference of $\Delta(.)$.

We also established the possibility of a coordination failure: when the efficient number of signals is 1, either firm generating one signal may be a Nash equilibrium, and in particular only the firm with the less informative signal generating information may be an equilibrium, which is inefficient.

Proof of Proposition 5

The pure strategy Nash equilibria of the information generation game for the case of a duopoly are similar to the ones derived for the case of a monopoly, modulo the different expression for the private benefit of information generation. We have:

- If $k > \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) (\Delta(\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is an equilibrium in which only firm i generates information.

- if $k \leq \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) (\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$, but $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise there is no information generation in equilibrium.

We follow the structure of the proof of Proposition 3 and consider different cases. For ease of notation let us define the social value of information generation as

$$S \cdot \Delta(\sigma, q_1, q_2) \equiv (E[\theta] - 2F(X^*)E[\theta|\theta < X^*]) \, \Delta(\sigma, q_1, q_2),$$

and the private value of information generation as

$$P \cdot \Delta(\sigma, q_1, q_2) \equiv \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2).$$

Condition (9) implies that $P > S$, so that the private benefit of information generation is higher than the social benefit. We distinguish three cases.

1. It is socially optimal to have no information generation, that is

$$k > S\Delta((\emptyset, \sigma_i), q_1, q_2) \text{ and } 2k > S\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

At least one firm i will invest if $k < P\Delta((\emptyset, \sigma_i), q_1, q_2)$, and thus the number of signals generated in equilibrium is higher than socially optimal if

$$S\Delta((\emptyset, \sigma_i), q_1, q_2) < k < P\Delta((\emptyset, \sigma_i), q_1, q_2) \text{ and } 2k > S\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

Otherwise, if the above condition does not hold, there may be an equilibrium, in which both firms invest, if

$$k \leq P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) \quad \forall i \in \{1, 2\}.$$

In this case there are, however, multiple equilibria: one with both firms investing and one with neither firm investing.

2. It is socially optimal for firm i to generate information but not firm $-i$, that is

$$S\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k < S\Delta((\emptyset, \sigma_i), q_1, q_2),$$
$$\Delta((\emptyset, \sigma_{-i}), q_1, q_2) < \Delta((\emptyset, \sigma_i), q_1, q_2).$$

Note that this case can only occur if $\Delta(.)$ has strictly decreasing differences in σ. Since $P > S$, at least one firm will invest in any Nash equilibrium, so the number of signals is at least the socially optimal one. For both firms to invest to be the unique Nash equilibrium it is necessary that

$$k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) \text{ and } k < P\Delta((\emptyset, \sigma_i), q_1, q_2),$$

Since $S < P$ the second condition holds. Hence, both firms will invest and there will be overinvestment if

$$S\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right).$$

If the above condition is violated, in equilibrium only one firm invests. If firm i invests then the equilibrium is efficient. If firm $-i$ invests, then the equilibrium is inefficient. In this last case, in equilibrium the information generated in equilibrium is *less* then the social optimum, because the firm with the least informative signal generates information in equilibrium.

3. It is socially optimal for both firms to generate information, that is

$$
\begin{aligned}
&2k < E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) \\
&E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k > E[\theta]\Delta((\emptyset, \sigma_1), q_1, q_2) - k \\
&E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k > E[\theta]\Delta((\emptyset, \sigma_2), q_1, q_2) - k
\end{aligned}
$$

   A necessary and sufficient condition for a Nash equilibrium, in which both firms generate information, is:

$$
k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),
$$

   for both firms $i = 1, 2$. Because $P > S$, there is always an equilibrium in which both firms generate information. Of course, there may also be another equilibrium, in which no firm generates information. But, as discussed in the text, when both equilibria are present the one in which both firms generate information Pareto dominates the other.

By restricting our attention to equilibria that are not Pareto dominated, we established that the number of signals drawn in equilibrium is always above the efficient one, strictly so in some cases. Also here, there is the possibility that the efficient number of signals is one, which is also the equilibrium one, but the "wrong" firm generates information in equilibrium.

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